Math 1411 **Practice Exam 2**

1) Let $f(x) = x^3 - 6x^2 + 15$
   a. Find the critical numbers of $f(x)$, if any.

   b. Find the open intervals on which the function is increasing or decreasing.

   c. Apply the First Derivative Test to identify all relative extrema, if any.

2) Let $g(x) = x(75 - x^2)$
   a. Find the critical numbers of $f(x)$, if any.

   b. Find the open intervals on which the function is increasing or decreasing.

   c. Apply the First Derivative Test to identify all relative extrema, if any.

3) Consider the function $f(x) = x^4 - 3x^3 + 4$
   a. Find the points of inflection(s), if any.

   b. Determine the intervals on which the function is concave up or concave down.

4) Consider the function $g(x) = \frac{x^2}{x^2+9}$
   a. Find the points of inflection(s), if any.

   b. Determine the intervals on which the function is concave up or concave down.

5) Determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$$f(x) = x^4 - 4x, \quad [0, 2]$$

6) A manufacturer wants to design a closed box having a square base and volume of 432 cubic inches. What dimension will produce a box with minimum surface area?

7) A rancher has 400 feet of fencing with which to enclose three adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?

8) A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$ .What length and width should the rectangle have so that its area is a maximum?

9) Find the following limits at infinity:

a) $\lim_{x\to\infty} \frac{2x-5}{\sqrt{x^2+3x}}$

b) $\lim_{x\to-\infty} \frac{2x-5}{\sqrt{x^2+3x}}$

c) $\lim_{x\to\infty} \frac{3-2x^2+5x^3}{4x^3-2x+1}$

10) Complete two iterations of Newton's Method for the function using the given initial guess.

$$f(x) = x^3 - 3, \quad x_1 = 1.4$$